Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Practice Paper 1

(Time: 1 hour 30 minutes) Paper Reference **9FM0/4A**

Further Mathematics

Advanced

Paper 4A: Further Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- The total mark for this part of the examination is 75. There are 8 questions.
- The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

1. Solve the congruence equation $17x \equiv 2 \pmod{75}$

(Total for Question 1 is 4 marks)

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2. P is the set of all prime numbers less than 20. A family of sets exist which all have four members $\{1, a, b, c\}$, such that $a, b, c \in P$ and $a < b < c$.

Colin deduces that there must be a finite number, N, of unique sets in this family.

(a) Find N.

(2)

Three possible members of this family of sets, denoted by $S\ A$, $S\ B$ and $S\ C$, are

$$S_A = \{1, 5, 7, 11\} \qquad S_B = \{1, 3, 7, 9\} \qquad S_C = \{1, 3, 5, 7\}$$

(b) Prove that the set S_A forms a non-cyclic group, G_A, under the binary operation of multiplication modulo 12.

[You may assume only that the law for associativity is already proven]

(5)

The set S_B forms a group, G_B, under multiplication modulo 10.
The set S_C forms a group, G_C, under multiplication modulo 8.

(c) Show that the group G_A is isomorphic to exactly one of the groups G_B or G_C.

(3)

Janet believes that when $a = 2$ in any of the sets belonging to this same family it is impossible for any such set to form a group under multiplication modulo n, where n is even.

(d) Explain why Janet is correct.

(2)

(Total for Question 2 is 12 marks)

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3. The point P represents a complex number z in an Argand diagram. Given that

$$\sqrt{2}|\, z - \mathrm{i}\, | = |\, z - 4\, |,$$

(a) find a Cartesian equation for the locus of P, simplifying your answer,

(3)

(b) sketch the locus of P.

(2)

(c) On your sketch from part **b**, shade the region for which

$$\sqrt{2}|\, z - \mathrm{I}\, | < |\, z - 4\, | \quad \text{and} \quad |\arg(z + 1)| < \frac{\pi}{2}.$$

(2)

(d) Find the complex numbers for which

$$\sqrt{2}|\, z - \mathrm{I}\, | = |\, z - 4\, | \text{ and } |\arg(z + 1)| = \frac{\pi}{2}.$$

(4)

(Total for Question 3 is 11 marks)

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4. $$\mathbf{M} = \begin{pmatrix} 1 & 0 & a \\ 0 & 2 & 0 \\ a & 0 & 0 \end{pmatrix}, \quad a \in \mathbb{R}.$$

For some value of $a > 0$, **M** has only two real eigenvalues.
One of these eigenvalues of **M** is -1.

(a) (i) Find the value of a.

(ii) Determine the second eigenvalue of **M** and justify which of the two eigenvalues is repeated.

(7)

M has three linearly independent eigenvectors.

The normalised eigenvector corresponding to the eigenvalue of -1 is $\begin{pmatrix} -\frac{1}{\sqrt{3}} \\ 0 \\ \sqrt{\frac{2}{3}} \end{pmatrix}$.

(b) Find the two remaining eigenvectors, giving your answers in normalised form.

(3)

(c) Write down a matrix **P** and a diagonal matrix **D** such that $\mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P} = \mathbf{D}$.

(2)

(Total for Question 4 is 12 marks)

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5. The total income, in pounds, of a charity in year n is denoted by $I_n = S_n + M_n + D_n$, where

Sn represents annual income from sales made in the charity's shops

Mn represents annual income from annual membership fees

Dn represents annual income from donations

The charity has formed three related models to try and predict its income in future years as follows:

$S_{n+1} = \frac{1}{6} I_n$

$M_{n+1} = 4S_{n+1} - S_n$

$D_{n+1} = d$, where d is constant

(a) Show that these models give rise to the overall recurrence relation model

$$I_{n+2} = \frac{5}{6} I_{n+1} - \frac{1}{6} I_n + d$$

(3)

(b) Given that $I_0 = d$ and $I_1 = \frac{7}{6}d$, find a closed form for I_n.

(6)

The charity states in its advertisements

'In the long term our ability to make a difference is entirely dependent on maintaining the value of the donations we receive.'

(c) Explain how the model supports this claim.

(2)

(Total for Question 5 is 11 marks)

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6. Consider the curve C generated by the parametric equations

$$x = (t-1)^2 \qquad \text{and} \qquad y = \frac{8}{3}t^{\frac{3}{2}}$$

An arc A of this curve C is defined by $0 \leq t \leq a$, where constant, $a > 0$. It is known that the arc length of A is 8.

(a) Find the value of a.

(5)

When this same arc A is rotated 360° around the y-axis, a curved surface is formed.

(b) Find the exact area of this curved surface.

(4)

(Total for Question 6 is 9 marks)

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7. Given that $I_n = \displaystyle\int_0^{\pi} \sin^{2n} x \ \mathrm{d}x, \quad n \in \mathbb{Z}, \quad n > 0,$

(a) establish the reduction formula

$$I_{n+1} = \left(\frac{2n+1}{2n+2}\right) I_n$$

(6)

Helen has developed the solution

$$\int_0^{\pi} \sin^{2n} x \ \mathrm{d}x = \frac{(2n)!\pi}{(n!)^2 2^{2n}}$$

(b) Given that $I_0 = \pi$, use the reduction formula to prove by induction that Helen's solution is valid.

(5)

(Total for Question 7 is 11 marks)

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8. Find the total number of positive integers less than 10 000 that contain the digit 7

(a) exactly once,

(2)

(b) at least once.

(3)

(Total for Question 8 is 5 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 2 IS 75 MARKS